Cross-like constructions and refinements

Daniel Soukup

The goal of this presentation is to give a brief preview on a special family of topological spaces on $\mathbb{R}^2$, which are originated in the idea of separate continuity.
Two main constructions are considered. Fix an $S \subseteq S^1$. The set $U \subseteq \mathbb{R}^2$ is an *S-radiolar open* iff for every point $x \in U$ it contains in every direction $s \in S$ a line segment covering x. Basic topological properties, such as separation axioms or the character of the topology are determined. We show that some properties, like connectedness, can be nicely characterized by simple non-topological properties of the defining set S. In favor of differentiating the S-radiolar topologies for different defining sets we investigate further properties, while the question whether two S-radiolar topologies are homeomorphic, in general, remains open.
While the first construction is non-regular for every nontrivial S, the second and more complicated constructions, denoted by $\mathcal{R}(S)$, will give Tychonoff spaces for closed S sets. As we will see when S is "nice" $\mathcal{R}(S)$ will be closely related to the Euclidean topology and will be hereditarily Lindelöf and separable. How ever, the problem of generally differentiating these topologies from each other is too unsolved.
My work in the topic in full detail can be downloaded from

`http://www.cs.elte.hu/blobs/diplomamunkak/bsc_mat/2009/soukup_daniel.pdf`